

Dr. Kamlesh Kumar
Asst. Prof. (Guest faculty)
Dept. of Mathematics
Maharaja College,
V.K.S.V, A-9

B.Sc. Part 1 (Hons) Paper 1

Rank of Matrix (Remaining part)

Q.2 Find the rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Soln.

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix} \quad (R_1 \leftrightarrow R_2)$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 3 & 7 \\ 3 & 4 & 9 & 10 \\ 6 & 9 & 12 & 17 \end{bmatrix} \quad \begin{array}{l} C_2 \rightarrow C_2 + C_1, \\ C_3 \rightarrow C_3 + 2C_1, \\ C_4 \rightarrow C_4 + 4C_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 3 & 7 \\ 3 & 4 & 3 & 10 \\ 6 & 9 & 4 & 17 \end{bmatrix} \quad C_3 \rightarrow \frac{1}{3}C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 7 \\ 0 & 4 & 3 & 10 \\ 0 & 9 & 4 & 17 \end{bmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - 6R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 7 \\ 0 & 3 & 4 & 10 \\ 0 & 4 & 9 & 17 \end{bmatrix} \quad C_3 \leftrightarrow C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & -11 & -11 \\ 0 & 0 & -11 & -11 \end{bmatrix} \quad \begin{array}{l} R_4 \rightarrow R_4 - 4R_2 \\ R_3 \rightarrow R_3 - 3R_2 \end{array}$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 - (-\frac{1}{11}) \\ R_4 \rightarrow R_4 - (-\frac{1}{11}) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} (C_2 \rightarrow C_2 - 5C_1) \\ (C_4 \rightarrow C_4 - 7C_1) \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} C_4 \rightarrow C_4 - C_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_4 \rightarrow R_4 - R_3 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

\Rightarrow Rank of $A = 3$

Q.3. Find the rank of $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Soln. $A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} (R_1 \leftrightarrow R_2) \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \end{bmatrix} \begin{matrix} C_3 \rightarrow C_3 - C_1 \\ C_4 \rightarrow C_4 - C_1 \end{matrix}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} \begin{matrix} R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} C_3 \rightarrow C_3 + (-\frac{1}{3}) \\ C_4 \rightarrow C_4 + (-1) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} C_3 \rightarrow C_3 + C_2 \\ C_4 \rightarrow C_4 - C_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3 \rightarrow R_3 - R_2 \\ R_4 \rightarrow R_4 - R_2 \end{matrix}$$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A \sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

\therefore Rank of $A = 2$